UCLouvain

Presentation

Adaptive regionalization for extreme precipitation

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Roadmap

Introduction

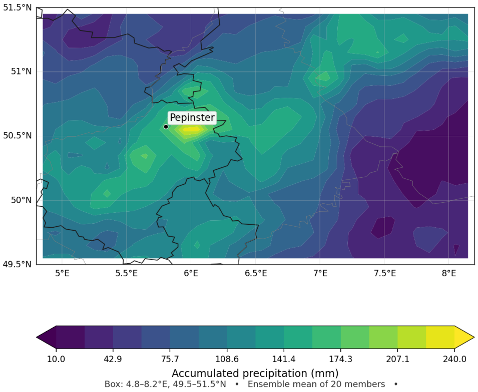
Adaptive regionalization for extreme precipitation

Context and Motivation

Extreme precipitation events across Europe.

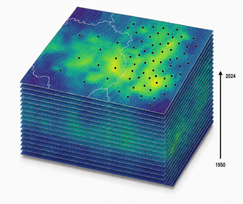
Interpolated observational data *(E-OBS dataset)*.

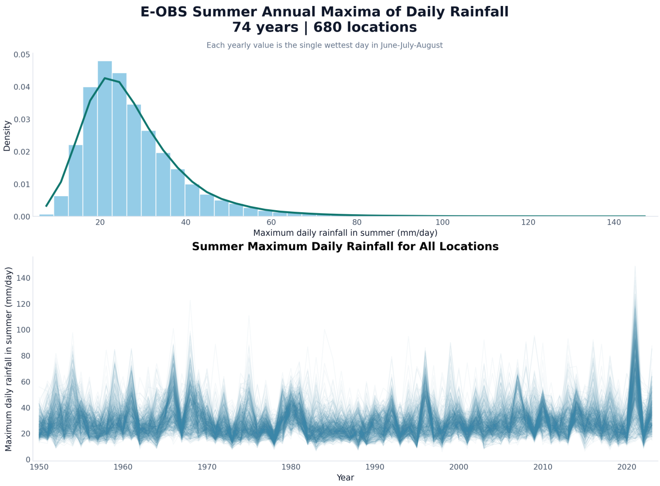
Precipitation Accumulation
(12-18 july 2021)



Space-time data

Yearly maxima of daily rainfall





Background

Adaptive regionalization for extreme precipitation

GEV model

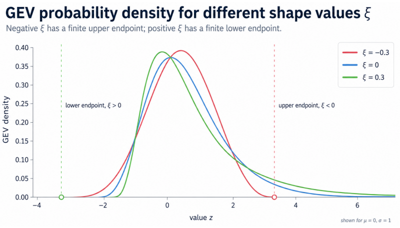
A flexible family for block maxima.

A random variable Z follows a GEV distribution with location $\mu \in \mathbb{R}$, scale $\sigma > 0$, and shape $\xi \in \mathbb{R}$ if

$$G(z; \mu, \sigma, \xi) = \exp\left\{ - \left[1 + \xi \left(\frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\},$$

for

$$1 + \xi \left(\frac{z - \mu}{\sigma} \right) > 0.$$



Support depends on ξ

Return Levels(1)

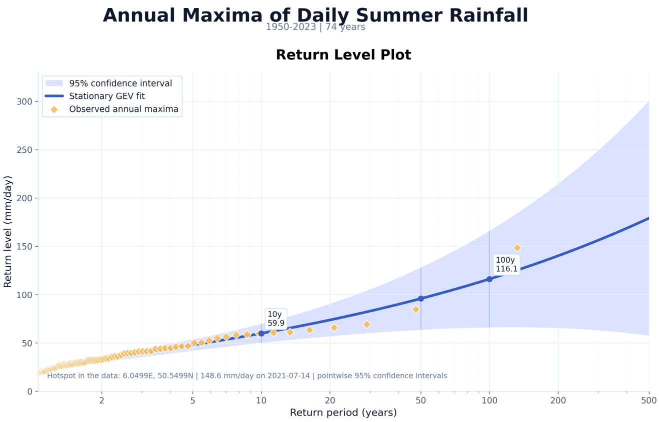
A return level is a **quantile** of the fitted GEV:

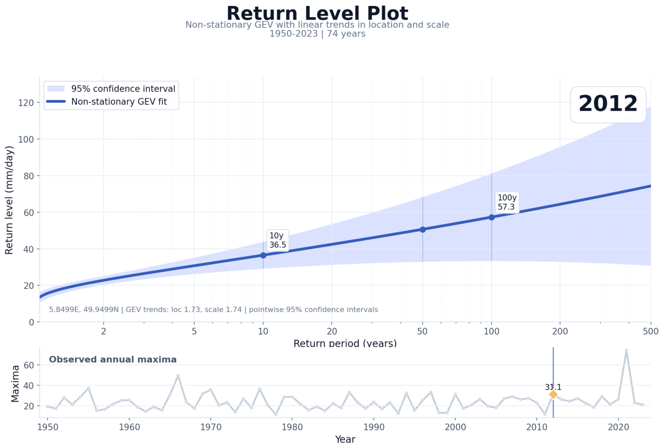
$$z_p = G^{-1}(p)$$

$$\mathbb{P}(Z \leq z_p) = p.$$

$$z_p = \begin{cases} \mu + \frac{\sigma}{\xi}\left([-\log(1-p)]^{-\xi} - 1\right), & \xi \neq 0, \\ \mu - \sigma \log\{-\log(1-p)\}, & \xi = 0. \end{cases}$$

z_p is the rainfall level associated with probability p.




Return Level Plot
Non-stationary GEV with linear trends in location and scale
1950-2023 | 74 years
95% confidence interval
Non-stationary GEV fit
2012
Return level (mm/day)
10y
36.5
100y
57.3
5.8499E, 49.9499N | GEV trends: loc 1.73, scale 1.74 | pointwise 95% confidence intervals
Return period (years)
Observed annual maxima
Maxima
31.1
Year

For a reference site s_0, we use the weighted independence likelihood

$$\ell_{s_0}(\boldsymbol{\beta};\mathbf{z}) = \sum_{\mathbf{s}\in\mathcal{S}} w_{s_0}(\mathbf{s}) \sum_{t\in\mathcal{T}} \log g(z_{t,\mathbf{s}};\boldsymbol{\theta}(t,\mathbf{s};\boldsymbol{\beta}))\,.$$

GEV parameterization.

$$\boldsymbol{\theta}(t,\mathrm{s};\boldsymbol{\beta}) = \big(\mu(t,\mathrm{s}),\sigma(t,\mathrm{s}),\xi\big).$$

$$\mu(t,\mathbf{s}) = \mathbf{x}_\mu(t,\mathbf{s})^\top \boldsymbol{\beta}^{(\mu)},$$

$$\sigma(t,\mathbf{s}) = \exp\left\{\mathbf{x}_\sigma(t,\mathbf{s})^\top \boldsymbol{\beta}^{(\sigma)}\right\}.$$

The weights $w_{s_0}(\mathrm{s})$ determine how much information site s contributes to the local fit at s_0.

Regionalization

Existing regionalization strategies

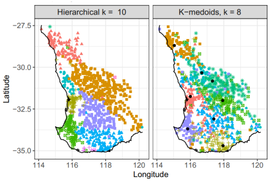
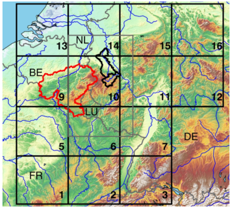


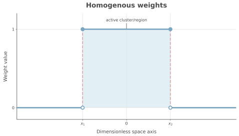
Fig. 6 Comparison of hierarchical clustering and K-medoids clustering for a set of stations in Southwest Western Australia

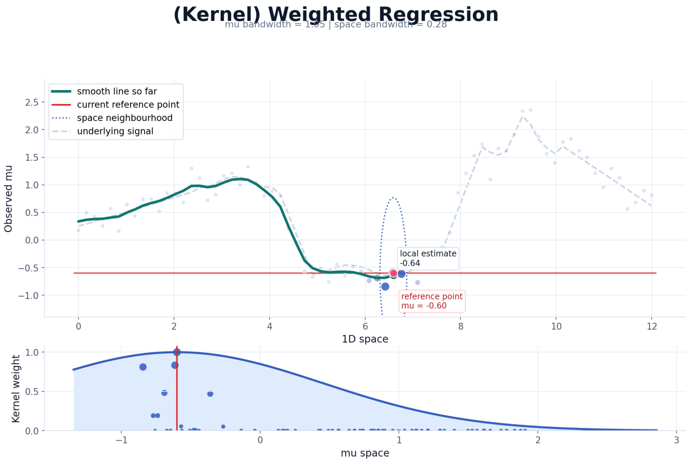
Tradowsky et al., 2023



Saunders et al., 2021

Weights for regionalization





(Kernel) Weighted Regression
mu bandwidth = 1.05 | space bandwidth = 0.28
smooth line so far
current reference point
space neighbourhood
underlying signal
Observed mu
1D space
local estimate
-0.64
reference point
mu = -0.60
Kernel weight
mu space

Methodology

Adaptive regionalization for extreme precipitation

From fixed kernels to learned weights

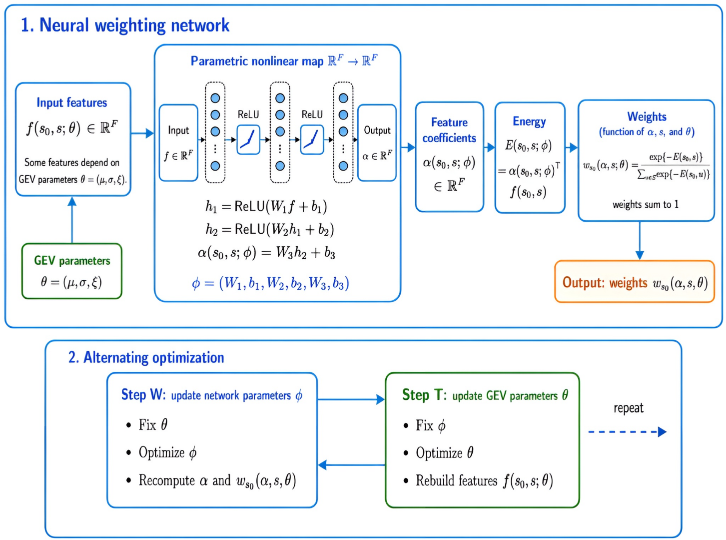
For a reference site $\mathbf{s}_0$ and a neighbouring site $\mathbf{s}$, we build a feature vector summarizing spatial and distributional distance

$$\Delta_\mu = \mu(\mathbf{s}_0) - \mu(\mathbf{s}), \qquad \Delta_{\log\sigma} = \log\sigma(\mathbf{s}_0) - \log\sigma(\mathbf{s}), \qquad \Delta_\xi = \xi(\mathbf{s}_0) - \xi(\mathbf{s})$$

$$\mathbf{f}(\mathbf{s}_0, \mathbf{s}) = \left[\frac{\|\mathbf{s}_0 - \mathbf{s}\|}{\gamma_d}, \frac{\Delta_\mu^2}{\gamma_\mu^2}, \frac{\Delta_{\log\sigma}^2}{\gamma_{\log\sigma}^2}, \frac{\Delta_\xi^2}{\gamma_\xi^2}, \ldots \right]^\top.$$

Interpretation: the first component measures **spatial separation**, while the remaining components compare **local GEV characteristics**. The constants γ put all features on comparable scales.

Architecture :-)



Optimization

We learn the weighting rule by maximizing the average local weighted likelihood

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\alpha}) = \frac{1}{|\mathcal{S}|} \sum_{\mathbf{s}_0 \in \mathcal{S}} \sum_{\mathbf{s} \in \mathcal{S}} w_{\mathbf{s}_0}(\mathbf{s}; \boldsymbol{\theta}, \boldsymbol{\alpha}) \left[\frac{1}{|\mathcal{T}|} \sum_{t \in \mathcal{T}} \log g(z_{t,\mathbf{s}}; \boldsymbol{\theta}(\mathbf{s}_0)) \right].$$

Optimization problem.

$$(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\alpha}}) = \arg\max_{\boldsymbol{\theta}, \boldsymbol{\alpha}} \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\alpha}).$$

The parameters $\boldsymbol{\alpha}$ define the feature coefficients, while $\boldsymbol{\theta}$ represents the local GEV parameters used inside the objective.

3-Step Method

1

Preliminary estimation with MLE

Obtain preliminary GEV parameters

$$\widehat{\boldsymbol{\theta}}(\mathrm{s}) = \big(\widehat{\mu}(\mathrm{s}), \widehat{\sigma}(\mathrm{s}), \widehat{\xi}(\mathrm{s})\big).$$

Pointwise or with spatial covariates $\boldsymbol{x}_\theta(\mathrm{s})$

2

Learn the weighting rule

Use the preliminary marginals to build input features to start the two-step optimization criterion.

3

Final estimation using MLE

Estimate the final local GEV parameters via MLE, with the weights fixed:
(Classical weighted likelihood optimization).

Simulations

Adaptive regionalization for extreme precipitation

Data Generation (1)

We simulate annual maxima on a spatio-temporal domain

$$(\mathbf{s}, t) \in \mathcal{S} \times \mathcal{T}, \qquad Z(\mathbf{s}, t) \sim \mathrm{GEV}(\mu(\mathbf{s}, t), \sigma(\mathbf{s}, t), \xi(\mathbf{s}, t))\,.$$

Covariate-driven parameter fields.

$$\begin{aligned}
\mu(\mathbf{s}, t) &= \beta_{\mu,0} + \beta_{\mu,s}\, \mathcal{C}_s(\mathbf{s}) + \beta_{\mu,t}\, \mathcal{C}_t(t), \\
\log \sigma(\mathbf{s}, t) &= \beta_{\sigma,0} + \beta_{\sigma,s}\, \mathcal{C}_s(\mathbf{s}) + \beta_{\sigma,t}\, \mathcal{C}_t(t), \\
\xi(\mathbf{s}, t) &= \xi_0 + \varepsilon_\xi(\mathbf{s}, t).
\end{aligned}$$

Normalized covariates.

$$\mathcal{C}(\cdot) = \frac{\widetilde{\mathcal{C}}(\cdot) - \min \widetilde{\mathcal{C}}}{\max \widetilde{\mathcal{C}} - \min \widetilde{\mathcal{C}}} \in [0, 1].$$

Spatial and temporal covariates induce smooth non-stationarity; $\varepsilon_\xi(\mathbf{s}, t) \in \{-0.05, +0.05\}$ is a weak mean-zero random perturbation.

Data Generation (2)

Temporal covariate.

$$\mathcal{C}_t(t) = \frac{t - t_{\min}}{t_{\max} - t_{\min}}, \qquad t \in \mathcal{T}.$$

Let $Z_s(\mathbf{s})$ be a mean-zero Gaussian random field. Its spatial covariance is taken from the Matérn family:

$$\mathrm{Cov}\{Z_s(\mathbf{s}), Z_s(\mathbf{s}')\} = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu}\, r_\phi(\mathbf{s},\mathbf{s}')\right)^{\nu} K_\nu\left(\sqrt{2\nu}\, r_\phi(\mathbf{s},\mathbf{s}')\right).$$

$$r_\phi(\mathbf{s},\mathbf{s}') = \left\| \begin{pmatrix} 1/\ell_x & 0 \\ 0 & 1/\ell_y \end{pmatrix} R_{-\phi}(\mathbf{s}-\mathbf{s}') \right\|.$$

Simulation parameters.

Parameter	**Intercept** (β_0)	**Spatial** (β_s)	**Temporal** (β_t)
Location μ	70.0	30	1.0
Scale σ	11	22	1.0
Shape ξ	0.1	0.0	0.0
ξ noise amplitude	$\delta_\xi = 0.05$	–	–

Evaluation Criteria

For each target quantity θ, performance is measured using the root mean squared error:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{r=1}^{N}\left(\widehat{\theta}_r - \theta\right)^2}.$$

Normalized error.
To compare quantities with different scales, we use the relative RMSE:

$$\mathrm{NRMSE} = \frac{\mathrm{RMSE}}{|\theta|}.$$

The normalization makes errors comparable across GEV parameters μ, σ, ξ and across return levels.

Results

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Effective Sample Size

A convenient summary of the amount of information used by a weighted procedure is

$$N_{\text{eff}} = \frac{\left(\sum_i w_i\right)^2}{\sum_i w_i^2}.$$

Normalized weights.

$$\text{If } \sum_i w_i = 1, \qquad N_{\text{eff}} = \frac{1}{\sum_i w_i^2}.$$

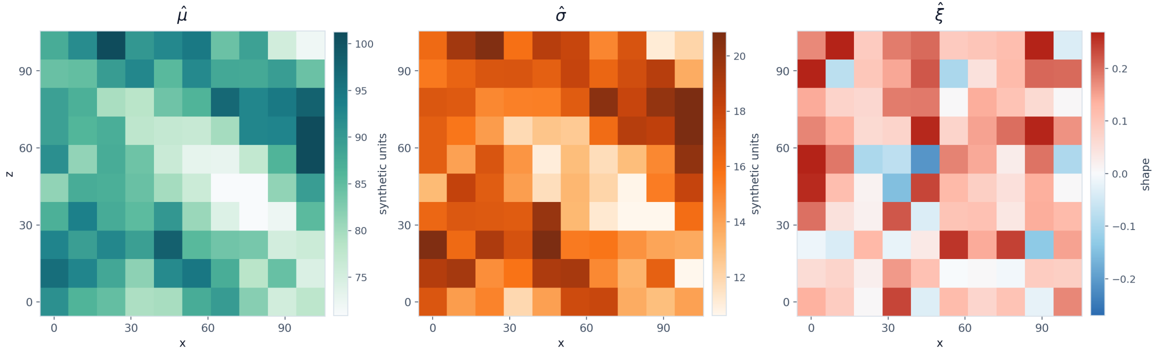
This is useful when the weights are interpreted as proportions of information: the more concentrated the weights are, the smaller N_{eff} becomes.

Uniform weights.

$$\text{If } w_1 = \cdots = w_n = 1, \qquad N_{\text{eff}} = \frac{n^2}{n} = n.$$

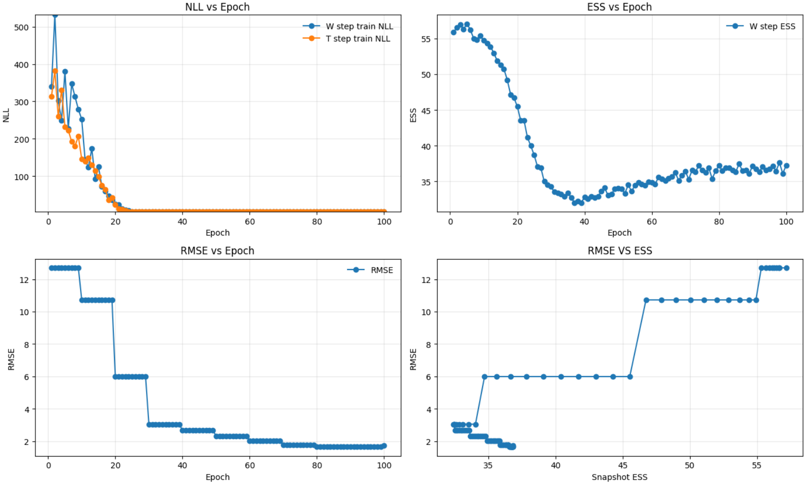
In our setting, ESS provides a quick indicator of how much information is effectively used after weighting.

Pointwise Marginal GEV Fit

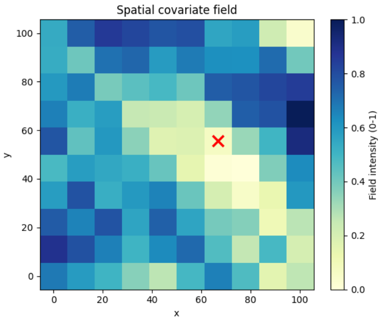


100 independent stationary fits on the SIM2 simulation field. Each site is fitted separately on its own 100-step series.

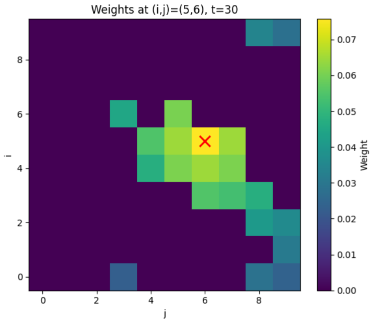
Step 2 : Gradient Descent



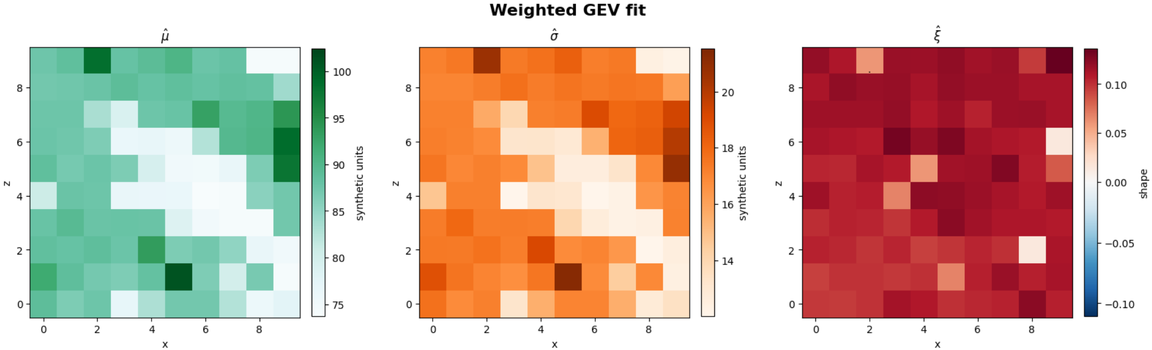
Learned Weights



The learned weights capture the structure of the spatial covariate field



Step 3 : Weighted fit



Comparing weighting methods

Table: Simulation performance summary. True values: $\mu = 76.08$, $\sigma = 12.66$, $\xi = 0.05$, $RL_{100} = 141.55$, and $RL_{1000} = 180.52$.

Metric	Parameter	Pointwise	Local	Weighted	Full
RMSE	μ	1.96	5.07	**1.83**	**8.66**
	σ	1.48	2.20	**1.15**	**4.46**
	ξ	**0.11**	0.05	**0.05**	0.06
	RL_{100}	21.21	26.82	**17.24**	**45.61**
	RL_{1000}	49.80	50.36	**35.74**	**81.15**
Bias	μ	**1.70**	5.06	1.81	**8.66**
	σ	**1.05**	2.19	1.13	**4.46**
	ξ	**0.05**	0.05	0.05	**0.06**
	RL_{100}	**16.50**	26.50	16.99	**45.57**
	RL_{1000}	**33.90**	49.10	34.98	**81.01**
Std	μ	**0.97**	0.39	0.23	**0.20**
	σ	**1.04**	0.19	0.22	**0.14**
	ξ	**0.10**	0.02	0.01	**0.01**
	RL_{100}	**13.33**	4.08	2.90	**1.82**
	RL_{1000}	**36.47**	11.22	7.31	**4.73**

WIP & Limitations

Adaptive regionalization for extreme precipitation

Discussion

Work in progress.

- Visualize the extension of the method to all reference sites
$$\mathbf{s}_0 \in \mathcal{S}.$$
Current simulation results focus on a single reference point.
- Determine when to stop the neural network training: validation set, early stopping criterion, or stability-based rule.
- Investigate feature selection more deeply: which features are truly informative for learning the weights?

Caveats.

- The main weakness is that the learned weights are data-dependent.
- As a result, the sandwich/Godambe estimator may underestimate uncertainty, leading to confidence intervals that are too narrow.

Thank you!

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Adaptive regionalization for extreme precipitation

Questions ?

References

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