

Introduction

- **Research question:**
To what extent does anthropogenic forcing increase the frequency, intensity, and duration of extreme heat events?
- **Method:** Compare CMIP6 climate simulations with and without anthropogenic forcing using space-time extreme value theory.
- **What do we focus on?**
Quantifying changes in heat wave characteristics over Europe.

Datasets

- **Observational data:** Daily maximum temperature from the Berkeley Earth dataset [2] (1°×1° resolution, 1880–present), combining over 30 000 weather stations with spatial reconstruction for exploratory analysis and clustering.
- **Model simulations:** CMIP6 climate models under **historical** (with human forcing, Factual) and **natural** (without human forcing, Counterfactual) runs (1850–2020).

Clustering region

- How: Partitioning around medoids.
- Distance based on pairwise tail dependence coefficient [3,4]:

$$d(\mathbf{s}_l, \mathbf{s}_k \mid u) = \frac{1 - \chi_{\mathbf{s}_l, \mathbf{s}_k, t_q, t_r}(u)}{2(3 - \chi_{\mathbf{s}_l, \mathbf{s}_k, t_q, t_r}(u))}$$

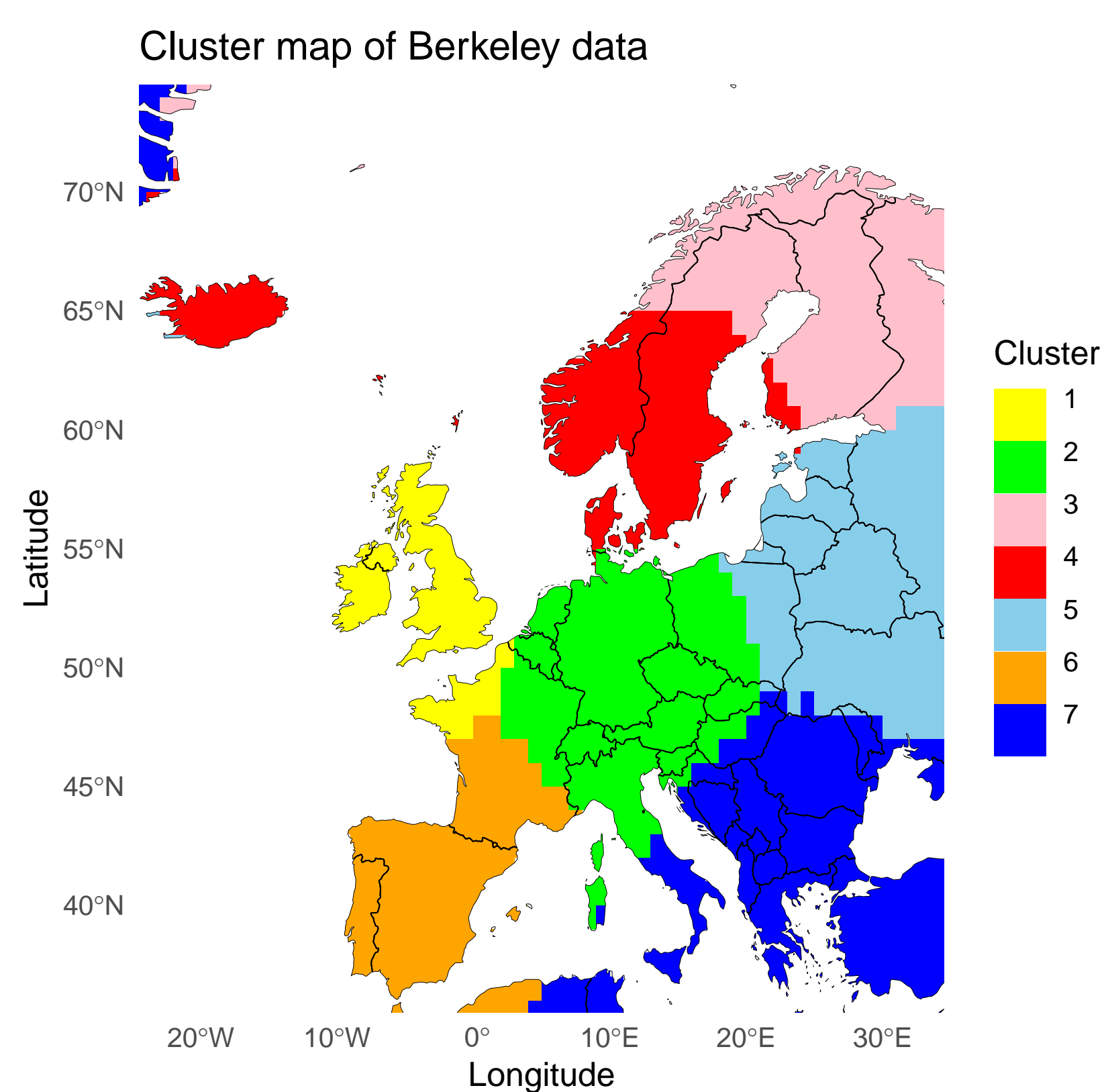


Figure 1. Cluster map of summer (JJA) temperature anomalies from the Berkeley dataset. The seven spatial clusters were identified using the partitioning around medoids (PAM) algorithm applied to a distance based on pairwise tail dependence.

Marginal model

The marginal distribution of each location $i \in \{1, \dots, n_j\}$ in a given cluster $j \in \{1, \dots, 7\}$ is modelled with a nonstationary generalized extreme value distribution:

$$G_{i,j}(z) = \exp \left\{ - \left[1 + \xi_{i,j} \left(\frac{z - \mu_{i,j}(t)}{\sigma_{i,j}} \right) \right]_+^{-1/\xi_{i,j}} \right\}$$

$$\mu_{i,j}(t) = \mu_{0,i,j} + \mu_{1,i,j} \text{GMST}(t) + \mu_{2,i,j} \text{ENSO3.4}(t) + \mu_{3,i,j} \text{NAO}(t)$$

where $\text{GMST}(t)$ is the annual Global Mean Surface Temperature anomaly, $\text{ENSO3.4}(t)$ is the July-centered 5-month running mean Niño 3.4 anomaly, and $\text{NAO}(t)$ is the standard-ized summer (JJA) North Atlantic Oscillation index.

We define the parameter vector for location i in cluster j as follows:

$$\beta_{i,j} := (\mu_{0,i,j}, \mu_{1,i,j}, \mu_{2,i,j}, \mu_{3,i,j}, \sigma_{i,j}, \xi_{i,j})^\top \in \mathbb{R}^6$$

We penalize variation across neighboring locations. Let $\gamma_{p,j} \in \mathbb{R}^{n_j}$ be the vector of the p -th parameter across all n_j locations in cluster j .

We estimate the parameters by minimizing a penalized negative log-likelihood [5,6,7]:

$$\hat{\beta}_j = \arg \min_{\beta_j} \left[-\log \mathcal{L}(\beta_j) + \sum_{p=1}^4 \lambda_{j,p} \gamma_{j,p}^\top \mathbf{S}_j \gamma_{j,p} \right]$$

where $\mathbf{S}_j \in \mathbb{R}^{n_j \times n_j}$ is an adjacency matrix:

$$s_{lk} = \begin{cases} |\mathcal{N}_l| & \text{if } l = k \\ -1 & \text{if } l \text{ and } k \text{ are neighbors} \\ 0 & \text{otherwise} \end{cases}$$

and \mathcal{N}_l denotes the set of neighbors of location l .

Spatio-temporal modeling

We model extremal dependence via the copula of a latent process [8,9] that is applied on all daily temperatures:

$$Z(\mathbf{s}, t) = R(t)^\delta W(\mathbf{s}, t)^{1-\delta}, \quad \delta \in [0, 1]$$

- $R(t)$: IID standard unit-Pareto
- $W(\mathbf{s}, t) = \frac{1}{1 - \Phi(W^*(\mathbf{s}, t))}$: space-time process
- $W^*(\mathbf{s}, t)$: zero-mean Gaussian field with separable covariance

$$C((\mathbf{s}_l, t_q), (\mathbf{s}_k, t_r)) = C_s(\mathbf{s}_l, \mathbf{s}_k) C_t(t_q, t_r)$$

Spatial covariance (Spatially nonstationary, anisotropic) [10]:

$$C_s(\mathbf{s}_l, \mathbf{s}_k) = \frac{\left(\sqrt{2} \theta_l^{1/4} \theta_k^{1/4} / (\theta_l + \theta_k)^{1/2} \right)^2}{1 + \left(\frac{\|A(\mathbf{s}_l - \mathbf{s}_k)\|}{\theta_l + \theta_k} \right)^2}$$

- $\theta_i \sim \text{Gamma}(\alpha, \tau)$: random local range
- A : anisotropy matrix

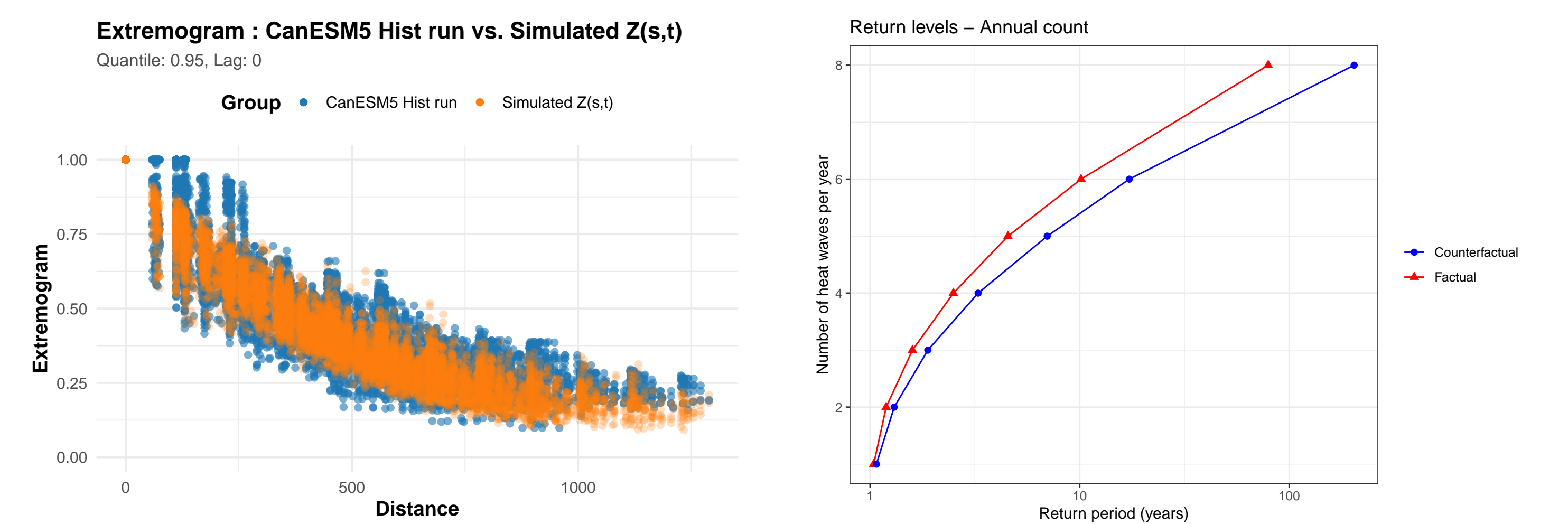
$$A = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix}$$

Temporal covariance [11]:

$$C_t(t_q, t_r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu} |t_q - t_r|}{\kappa} \right)^\nu K_\nu \left(\frac{\sqrt{2\nu} |t_q - t_r|}{\kappa} \right)$$

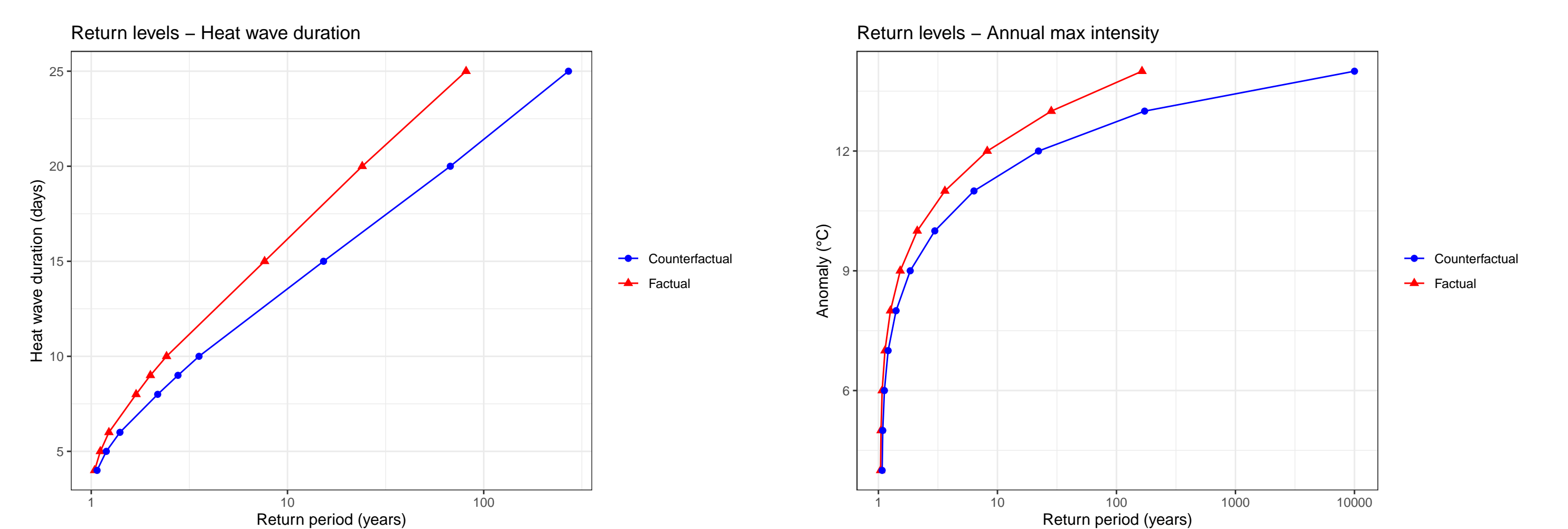
- Likelihood-free parameter estimation with neural Bayes estimators [12].

Preliminary results on CanESM5 model [13] **cluster 1**



(a) Empirical extremogram at lag 0 as a function of spatial distance Cluster 1. Results from historical data and simulations of the $Z(\mathbf{s}, t)$ process are shown for comparison.

(b) Return levels of heat wave number during a summer Cluster 1.



(c) Return levels of heat wave duration Cluster 1.

(d) Return levels of heat wave intensity Cluster 1.

Figure 2. Preliminary results on CanESM5 model cluster 1. Historical (Factual) and natural (Counterfactual) scenarios are shown for comparison.

References

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